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Some Dynamic Effects in High-Speed Solid-Lubricated Ball Bearings

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Dynamic performance simulations of a high-load, high-speed ball bearing for turbine-engine applications are considered using the available Dynamics of Rolling Element Bearings (DREB) computer program. It is shown that the key element in the bearing design is the traction behavior at the ball/race interface for the prescribed materials. With a given traction model, the geometry of the bearing may be designed to ensure acceptable ball/cage collision forces and to ensure the general stability of the cage.

INTRODUCTION

The increasing operating temperatures and speeds have generated considerable interest in the development of solid-lubricated bearings for advanced turbine-engine applications. Since the primary components which are influenced by the lubrication mode are the bearings supporting the main engine shaft and other support and guidance systems, the problem has been directed towards the development of solid-lubricated bearings. Among a number of different possible bearings or bearing systems, rolling-element ball bearings have been selected for the present investigation. Thus, the objective of the current work is to investigate the feasibility of solid-lubricated ball bearings for high-load and high-speed turbine-engine applications.

The key to an acceptable bearing performance lies in the tractive behavior of the lubricating material and the resulting heat generation in the bearing. It is, therefore, necessary to establish a relationship between the overall bearing performance and the lubricant traction behavior. Also, the determination of the traction characteristics for the candidate materials or the development of materials for certain traction behavior is essential. The present investigation is directed towards bearing performance simulations only and it, therefore, constitutes only one part of the twofold problem.

Over the past decade, the advent of high-speed computing systems has led to a noted advancement in the performance simulation of rolling bearings. The simple static or quasi-static models of Jones (*1*), (*2*) have been advanced to generalized dynamic models of Gupta (*3*), (*4*) and hence a real time-performance simulation of rolling bearings has become possible. Such analytical simulations have proven to be very efficient for scanning the bearing behavior over an extensive range of operational and design parameters. However, the sophistications associated with such analytical models, and the assumptions made on the fundamental level, do warrant experimental validation of the analytical predictions at least at some selected conditions. Analytical simulations, at this point, will, therefore, only provide answers to feasibility questions and may make certain design recommendations but the final design will be a close iteration between the analytical simulations and experimental evaluation of the bearings. Hence, the results presented in this paper should be viewed only as a first step towards the ultimate objective of solid-lubricated rolling bearing development.

A truly dynamic model for simulating the motions of the balls, cage, and races of a ball bearing, being the most advanced model available to date, is selected as the basis for the present investigation. A Dynamics of Rolling Element Bearings (DREB) computer program (*5*) based on the generalized dynamic formulation (*3*) is modified to model solid-lubricated ball/race interaction and the modified program is then used to obtain the results presented herein. The comparison of the capabilities of DREB with other models is omitted here for brevity; however, Refs. (*6*) and (*7*) contain extensive reviews of the available computer programs for rolling bearing modelling and analysis.

Before discussing the performance simulations of the turbine-engine bearing, some details of the solid-lubricated ball/race interaction are presented in the following section.

SOLID-LUBRICATED BALL/RACE INTERACTION

The available DREB computer program (*5*) is primarily designed for liquid-lubricated bearings and the ball/race interaction is determined by the elastohydrodynamic behavior of the lubricant. On a local scale, the contact ellipse

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is divided into several incremental strips along the rolling direction and the net traction is determined by summation over the incremental strips. This consists of a simple one-dimensional integration if the variation of traction over the incremental strip is allowed only in an integrated fashion (*8*). Although such a procedure has been acceptable for elastohydrodynamic conditions, solid lubrication will warrant changes in both the traction-slip relation and the integration algorithm. For a realistic simulation, it will be necessary to replace the one-dimensional integration scheme with a two-dimensional algorithm for determining the net traction over the elliptical contact zone. Thus, for any prescribed traction-slip relation, the variation of traction coefficient over the entire contact zone should be considered. DREB is, therefore, modified on the basis of the following analytical foundation.

Possible Traction-Slip Models

The high-load and high-speed operating environment of a turbine engine demands that lubrication at the ball/race interface be provided in a replenishable fashion. Thus, the cage is made of a solid lubricant and, as a result of ball/cage collision, the lubricant is released and transferred to the ball/race interface in the form of a sacrificial layer or a transfer film. For such a process, not only is the amount of experimental data on the traction behavior at the ball/race contact very limited, but considerable research is necessary to develop satisfactory materials for the cage. To date, a Ga/In/WSe_2 composite has been used with some success. Recently obtained experimental data (*14*) on the traction behavior of this material is shown in Fig. 1. The solid lines indicate a reasonable approximation to the experimental data and it will, therefore, be used in investigating the bearing performance under the simulated operating conditions.

Numerical Integration Algorithm

Once the traction slip relation is known, the traction coefficient at any point in the contact ellipse can be determined since the slip velocity at the point is prescribed. The normal pressure or the load on an incremental area around the point is given by the Hertzian solutions and the net traction force can be determined by integrating the incremental traction over the area of the contact zone. Since the variation of pressure is of the type $\sqrt{1 - r^2}$, it cannot be approximated by a polynominal, specially near the boundary of the contact zone where $r \rightarrow 1$. Therefore, conventional Gaussian quadrature formulae (*11*), (*12*) for two-dimensional integration cannot be used economically and considerable care must be exercised in computing the required integral. The following algorithm is based on Chebyshev polynominals which are best suited to the current problem.

As shown in Fig. 2(a), the traction force f at any point will be a function of the position coordinates (x, y) with the dimensional coordinates:

$$\xi = \frac{x}{a}, \text{ and } \eta = \frac{y}{b} \qquad [1]$$

the elliptical contact zone can be transformed into a unit circle [see Fig. 2(b)] and the traction integral may be written as

$$\begin{aligned} \vec{\mathcal{F}} &= \iint \vec{f}\,(x, y)\, dx\, dy \\ &= \int_0^1 \int_0^{2\pi r} \vec{f}\,(r \cos \theta, r \sin \theta)\, ds\, dr \end{aligned} \qquad [2]$$

At any point, P in the contact zone the Hertzian pressure will vary as $\sqrt{1 - r^2}$ and, therefore, the incremental traction force function may be written as

$$\vec{f}\,(r \cos \theta, r \sin \theta) = \sqrt{1 - r^2}\, \vec{F}\,(r \cos \theta, r \sin \theta) \qquad [3]$$

Combining [2] and [3] gives

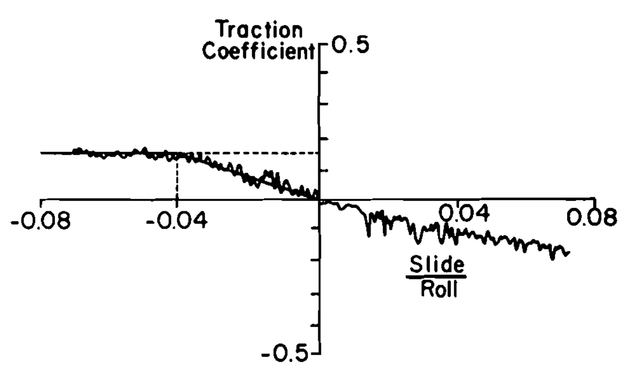

Fig. 1—Traction behavior of a 0.40 μm coating of Ga-In-W-Se_2 on 52100 steel specimens at a contact pressure of 1.07 GPa, temperature of 26°C and rolling speed of 2 m/s After 2800 revolutions of the test specimen (14).

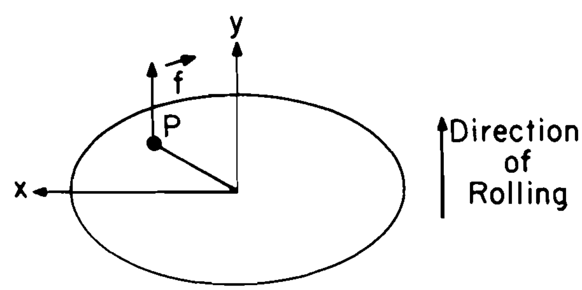

Fig. (a)—contact zone at the ball/race interaction

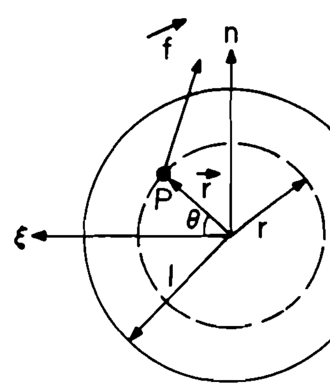

Fig. (b)—elliptical contact zone transformed into a unit circle

Fig. 2—A contact zone transformation at ball/race interaction

$$\vec{\mathcal{F}} = \int_0^1 \sqrt{1 - r^2} \int_0^{2\pi r} \vec{F}\,(r \cos \theta, r \sin \theta)\, ds\, dr$$

Replacing the inner integral with a summation (*13*) will give the final formula

$$\vec{\mathcal{F}} = \int_0^1 \sqrt{1 - r^2}\, \vec{F}_r\, dr \qquad [4]$$

where

$$\vec{F}_r = \frac{\pi r}{m} \sum_{i=1}^{2m} \vec{F} \left(r \cos \frac{\pi i}{m}, r \sin \frac{\pi i}{m} \right) \qquad [5]$$

and m is the number of equally spaced points on the circle of radius r.

Since F_r can be approximated by a polynomial, Eq. [4] represents a formula similar to the classical Chebyshev integration formula except that the limits of integration are 0 to 1 as compared to the conventional limits of -1 to 1.

Several orders of polynomials are considered in Eq. [4] and it is found that a fifth order approximation will be adequate for the present problem. Also, m is arbitrarily selected as 72. This will give us a point on the circle at every 5° and hence the variation of slip along the circle can be treated with adequate accuracy. Thus, the final integration algorithm written in terms of Chebyshev weights and the roots of the polynominal is

$$\vec{\mathcal{F}} = \sum_{j=1}^{n} \omega_j \left\{ \left(\frac{\pi r_j}{m} \sum_{i=1}^{2m} \vec{F}\, (r_j \cos \frac{\pi i}{m}, r_j \sin \frac{\pi i}{m} \right] \right\} \qquad [6]$$

where m and n are selected to be as 72 and 5, respectively, and the roots r_j and weights ω_j are computed to be as follows:

j	r_j	ω_j
1	0.0445594672	0.112314749
2	0.218694082	0.220987601
3	0.474310986	0.239278553
4	0.735889118	0.161581619
5	0.929067427	0.0512356414

Note that $\vec{F}$ in equation [6] is the local traction vector, which is related to the local slip vector via the traction slip relation.

The above algorithm is incorporated in the DREB computer program to produce the results discussed in the next sections.

TURBINE ENGINE BEARING PERFORMANCE

With the assumption that lubrication at ball/race interactions is provided by replenishable transfer film formed by the material released from the cage pockets as a result of ball/cage collisions, the key elements in the bearing performance simulation are the examination of the nature of ball/cage collisions and the magnitude of the collision forces. Ball/cage interaction will, therefore, be emphasized in the results presented below for a prescribed bearing design.

Bearing Design

Table 1 lists the geometrical and materials parameters for a turbine-engine bearing. The cage pockets are assumed to be made of the lubricating material such as the previously mentioned composite ($Ga/In/WSe_2$). However, since little is known about the elastic properties of this material, the elastic modulus and Poisson's ratio for the computation of ball/cage force are assumed to be the same as that of steel.

Operational Parameters

The bearing is assumed to be operating at 63 500 rpm, which is typical of high-speed turbine engines. Also,

TABLE 1—BEARING GEOMETRY

Bore	=	29.542 mm
Outside Diameter	=	47.00 mm
Ball Diameter	=	5.556 mm
Number of Balls	=	17
Pitch Diameter	=	38.51 mm
Contact Angle	=	18.60°
Outer Race Curvature Factor	=	0.52
Inner Race Curvature Factor	=	0.54
Cage Outside Diameter	=	41.224 mm
Cage Inside Diameter	=	37.389 mm
Cage Width	=	8.191 mm
Type of Guidance	=	Outer
Diametral Cage/Race Clearance	=	0.330 mm
Cage/Ball Pocket Clearance	=	0.457 mm
Elastic Modulus for Balls and Races	=	2.0×10^{11} N/M^2
Elastic Modulus for Cage	=	2.0×10^{11} N/M^2
Poisson's Ratio for Balls and Races	=	0.25
Poisson's Ratio for Cage	=	0.25
Material Density for Balls and Races	=	7.75×10^3 kgm/M^3
Material Density for Cage	=	7.92×10^3 kgm/M^3

a thrust load of 400 N is applied on the bearing. At such a high-speed operation, it is very likely that the bearing will see an unbalance load synchronous with the rotating shaft. Thus, a rotating load of 80 N is assumed to be applied on the bearing. Such a loading will cause a whirl of the inner race with a radius (i.e., the radial deflection) of 0.809 micron, while the outer race is held fixed. The axial position of the inner race is held fixed corresponding to an equilibrium position for the 400 N thrust load.

Bearing Performance Simulations

With the above operating conditions, a quasi-static equilibrium solution is obtained to determine the initial conditions and the cage mass center is assumed to coincide with the bearing center at time zero. The imposed radial motion on the inner race resulting from a rotating load is shown in Fig. 3, which also shows the real time interval (corresponding to more than a shaft revolution) over which the bearing performance is simulated.

As a first case, the bearing design summarized in Table 1 is considered, and, using the modified DREB computer program, the dynamic performance of the bearing is simulated over almost two shaft revolutions. With the prescribed geometry and with the ball/race traction model of Fig. 1, it is found that the ball excursions are much smaller compared to the ball pocket clearance and, hence, no significant ball/cage collisions are observed. Such a condition is clearly unacceptable because it would result in no lubricant transfer from the cage. Hence, the present geometry, which is meant for an oil-lubricated bearing, may not be acceptable for solid-lubrication environment where frequent ball/cage collisions are required to extract the lubricant from the cage.

In the second case, the ball-pocket clearance is reduced to 0.20 mm and the performance simulation is repeated using the DREB computer program. With this reduced pocket clearance, a number of collisions are observed, as seen in Fig. 4. Maximum ball/cage force of about 28 N is observed. As a result of the ball/cage collisions, the cage does develop a whirl of about half its angular velocity as seen in Fig. 5. The bearing power loss and race torque variations are shown in Fig. 6. The peaks correspond to ball/cage collisions. Nominal value of observed race torque is about 0.03 NM and the power loss is about 280 watts. All these performance parameters may be quite acceptable, except that more frequent ball/cage collisions than those

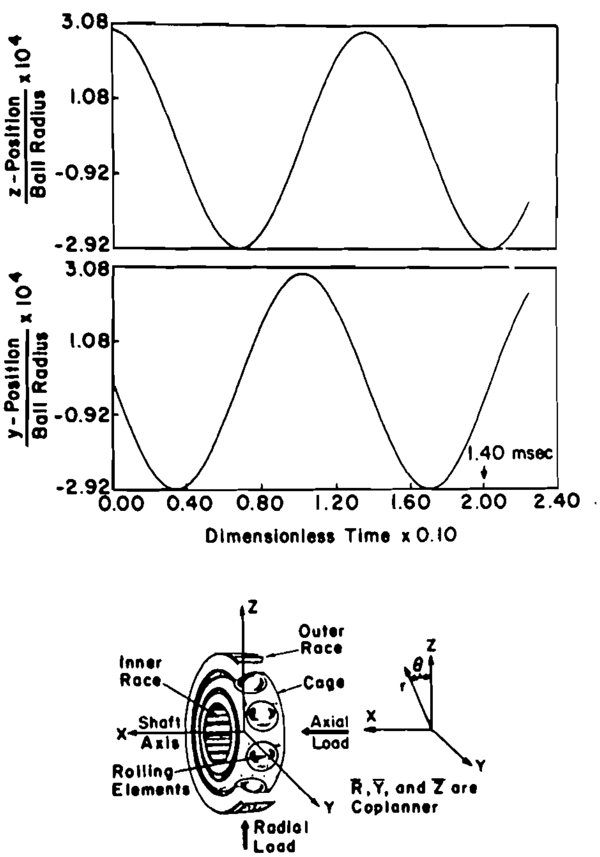

Fig. 3—Imposed inner race motion on the test bearing

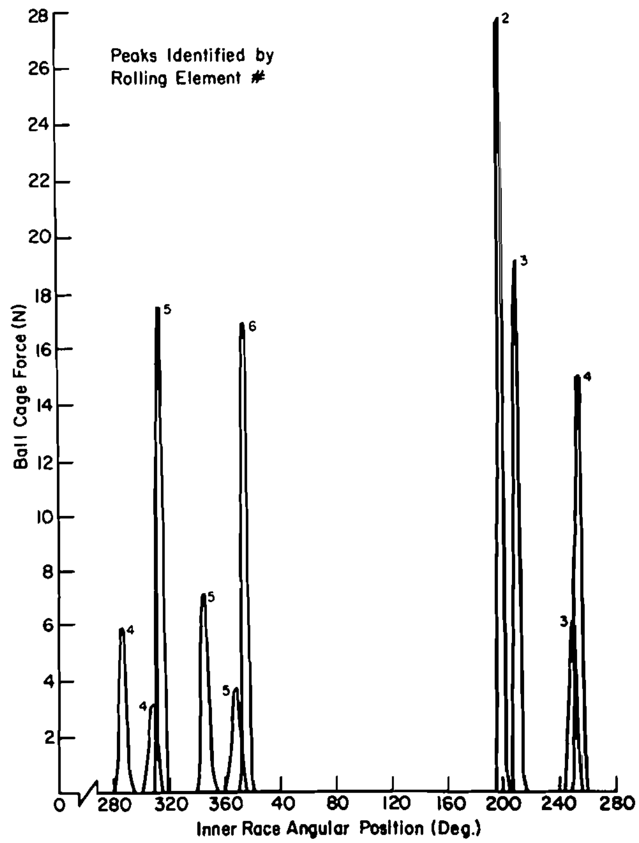

Fig. 4—Ball/cage collisions with a pocket clearance of 0.2 mm and ball/race traction model of Fig. 1.

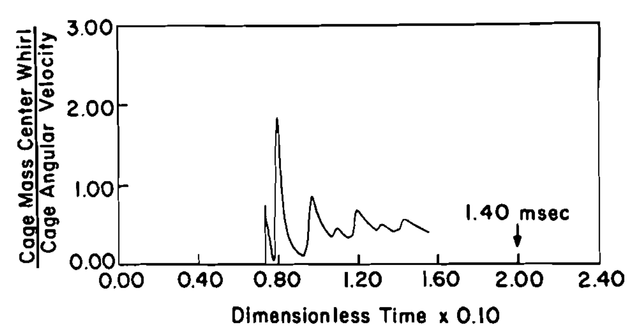

Fig. 5—Cage mass center whirl resulting from ball/cage collisions of Fig. 4.

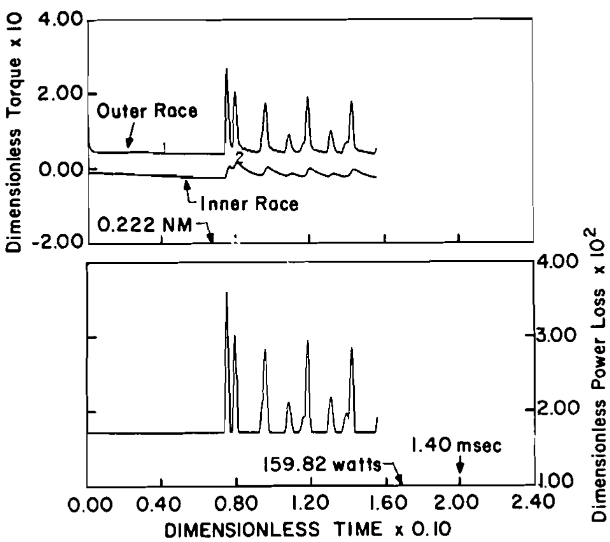

Fig. 6—Bearing power loss and torque behavior with the traction model of Fig. 1.

seen in Fig. 4 may be desirable to ensure a continued transfer of lubricant to the ball/race interface.

With the forementioned requirement of more frequent collisions, a third case is simulated, where the traction model of Fig. 1 is replaced by that of Fig. 7. It may be noted the model shown in Fig. 7 is not only different in nature but the magnitude of traction coefficients are also very low. Although the selection of such a model is mostly arbitrary at this point, some justification for its nature has been reported earlier (*9*).

The lower values of traction coefficient are selected basically to provide increased ball slippage and more frequent ball/cage collision. It may very well be possible that at higher temperatures (~400°C), which may be more realistic for engine application, the traction coefficients shown in Fig. 1 (at 26°C) may be greatly reduced. It has been shown that, for certain coatings, the friction at high temperatures is almost an order of magnitude lower than that at room temperature (*10*). Hence, the selection of the traction model shown in Fig. 7 may not be all that unrealistic.

The bearing performance simulations are repeated with the substitution of the above traction model. The observed ball/cage collision pattern is shown in Fig. 8. It is seen that although the general magnitude of the ball/cage forces is not greatly different from that observed earlier, the frequency of collisions is certainly greater. It can, therefore, be proven that a close iteration between the ball/race traction model and the ball pocket clearance can lead to an acceptable bearing design.

With the substitution of the hypothetical traction model of Fig. 8, the cage whirl characteristics are relatively unchanged but the power loss and race torque values are greatly reduced due to substantially lower traction. A nominal power loss of about 16 watts and race torque of about 0.004 NM is observed.

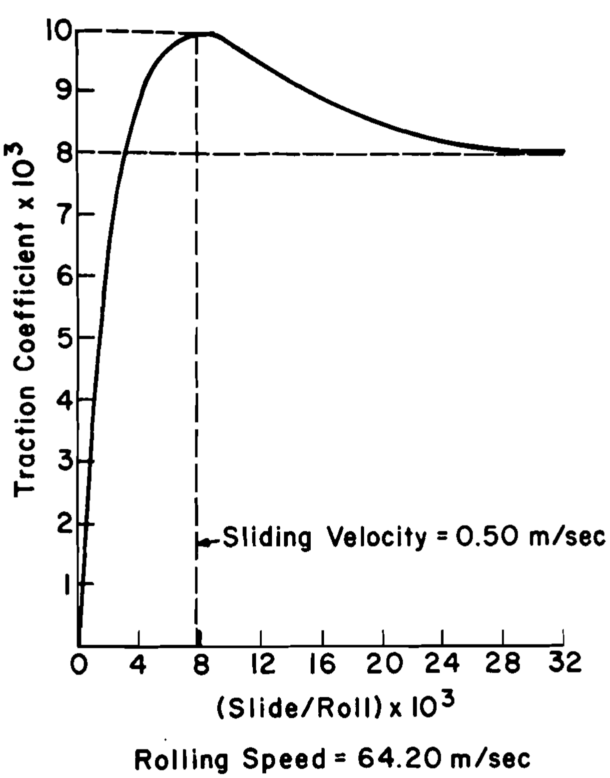

Fig. 7—A hypothetical traction model for the ball/race interaction

CONCLUSIONS AND DESIGN RECOMMENDATIONS

From the analytical simulation of the dynamic performance of a typical engine ball bearing under solid-lubrication conditions, it is seen that the ball/race traction model is a key element in the bearing design. If the lubrication mechanism consists of a transfer film formation at the ball/race interface, following the release of the lubricating material from the cage as a result of ball/cage collision, then the ball-pocket clearance will strongly couple with the traction model. An optimum design will consist of a close iteration between the traction behavior and the ball-pocket clearance.

Based on the foregoing conclusions, the following recommendations may be made for acceptable designs of solid-lubricated ball bearings:

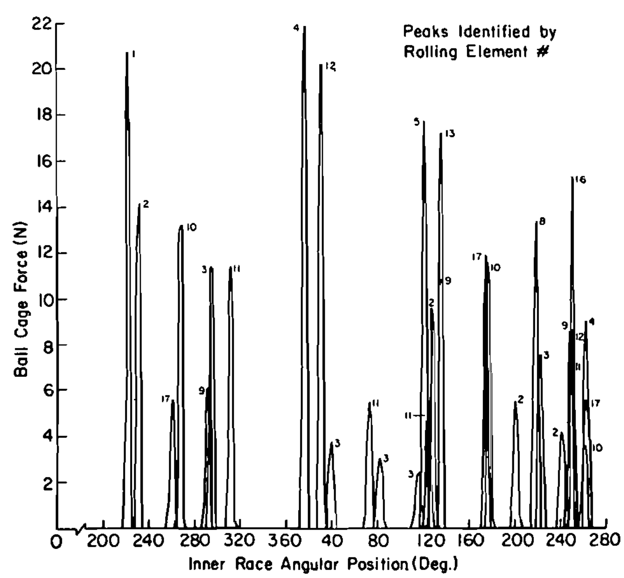

Fig. 8—Ball/cage collision forces with a pocket clearance of 0.20 mm and traction model of Fig. 7.

1. Investigate the traction behavior of candidate materials under the range of sliding speeds and operating temperatures existing in the bearing.
2. Carry out transfer film studies to determine the normal force required, at the ball/cage interface, to produce adequate material for the required transfer film.
3. With the knowledge of the traction behavior and the normal load required to produce the transfer film, bearing geometry should be optimized parametrically. The key performance parameters will be the ball/cage collision force, which should compare with the required force to establish the desired transfer film, and the general stability of the balls and the cage over the range of operating conditions.

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DISCUSSION

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Dr. Gupta's paper presents an interesting evaluation of solid-film bearing dynamics. The paper shows that from the point of view of cage stability, a solid-film bearing may behave in a manner similar to a liquid-lubricated manner. The paper also shows that the traction model for the ball-race contact, used for the calculations, may have only secondary effects on bearing dynamics. Since ball-race traction is a primary input to cage-dynamics calculations, this lack of sensitivity of dynamics to traction model seems like a non sequitur to the reader.

In recent bearing dynamic studies here (*1*), (*2*), a cage stability criterion has been developed. This criterion is based on a dynamic force balance between the ball-cage and ball-race contact. The critical parameter for the calculations is a damping term defined by

$$D_p = \frac{32C_\mu{}^2}{M\,C_{s1}} \quad . \qquad \text{[A1]}$$

If $D_p < 1$, the cage is stable, where

M is the cage mass

$$\left(\approx 5.9\times 10^{-5}\ \text{lb-s}^2/\text{in or } 10^{-2}\,\frac{\text{N-s}^2}{M}\right)$$

C_{s1} is the ball-cage spring rate

$(\approx 3.4 \times 10^5$ lb/in or 6×10^7 N/M)

C_μ is the ball-race damping term which could be defined as

$$C_\mu = F/\Delta V \quad ,$$

where F is the traction at a slip of ΔV.

Using Figs. 1 and 7 from Gupta's paper and assuming reasonable ball loadings,

C_μ (Fig. 1) = 0.84 lb-s/in or 146 N-s/M
C_μ (Fig. 7) = 0.026 lb-s/in or 4.6 N-s/M.

For the model of Fig. 7, $D_p = 10^{-3}$ which implies that the ball-race contact completely absorbs the dynamic cage motions and the cage should be stable. For the model of Fig. 1, $D_p = 1.2$ which implies some rebounding of the cage after a ball cage impact. The amount of rebounding is given by the equation

$$e_c = \exp\left(\frac{-\pi}{\sqrt{D_p - 1}}\right) = 10^{-4} \quad . \qquad \text{[A2]}$$

Using Fig. A1 from Ref. (*A1*), it can be seen that even with the Fig. 1 model, the cage is still stable for any reasonable value of the ball-cage friction coefficient.

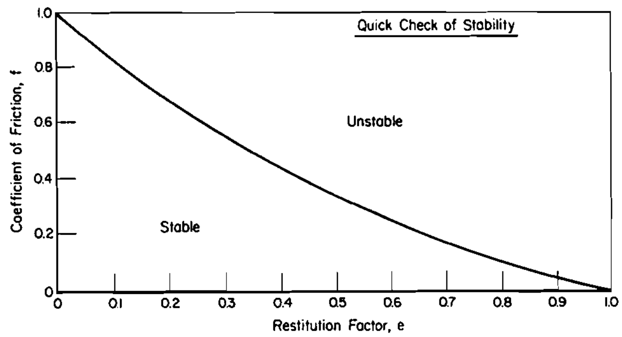

Fig. A1—Typical prediction for BASDAP computer model

Therefore, even though the data of Figs. 1 and 7 yield different damping parameters, they both imply stable bearing operation. The implication is that a solid-film bearing can be stable. This is not to imply that any dry film bearing would be stable, but rather that solid films conforming to the traction model described in the paper would be stable. Another traction model could easily produce instabilities; for example, if in the model of Fig. 1, the traction coefficient had risen to 0.16 at a slide/roll motion of 0.004 rather than 0.04, then D_p would be 120, and an unstable situation would be likely.

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DISCUSSION

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Prohibitively increased lead time and cost of developing new mechanical assemblies in the model shop render design-by-software a necessary tool. However, because of proprietary smoke screens or due to the fog of our own ignorance about programming, most of us tend to be cautiously hopeful but still somewhat skeptical of computer diagnostics' high priests.

To be fair, burning only a few ounces of midnight oil can reveal even to the most old-fashioned compupeasant tribologist that the quasistatic bearing models have, indeed, been helpful for aiding bearing design or improving bearing performance. However, depending on the initial simplifying assumptions that do vary from model to model, as well as on the mechanical and mathematical skill of the respective programmer to properly weave the subroutines, the consistency of their prediction is yet inadequate to serve in a capacity other than a sheep-from-the-goats separation device.

Fully dynamic models, such as Dr. Gupta's DREB could do a lot more—theoretically. Current practical considerations, e.g., prohibitively long computer time needed to iterate a large number of equations for an adequate number of bearing revolutions, inability to handle anything more than a symmetrical retainer ring or thick and unwieldy "books" of computer statements prevent the dynamic models to live up to their expectations. This is true even for oil-lubricated bearings, which have seen software massage for some time now, without producing any more than a trace of correlation between prediction and practice.

Considering the rapid advances in bearing and computer technology, these shortcomings are only temporary. We must not hesitate to increase the power of all computer diagnostic techniques, especially the dynamic models, to produce unique bearing designs. The largest challenge is now waiting in the wings: solid-lubricated rolling element bearings for extreme environment, high-power density applications. Dr. Gupta's paper is a first step toward such a bearing designed from scratch, based on fundamental considerations alone. His work clearly indicates a far greater need for cooperation among the computer technologist, the materials scientist, and the test rig engineer than ever before existed.

New work like this, of the preliminary nature, defies criticism—for now. There are a few questions, however, that should be clarified at this time:

1. I have yet to see a solid lubricant traction curve which did not exhibit a positive slope crossing the origin (i.e., at the point of pure rolling). Does that mean that all solid lubricant bearings should exhibit minimum instability as far as traction value requirements are concerned? This discusser is skeptical about the true traction—speed variation of any solid lubricant in the Hertzian contact zone (i.e., on the microscale), as measured by macro-type traction testers. Perhaps the size of the slope may prove to be a more useful criterion of stability than the simple fact that it is positive.
2. Reducing ball-to-ball pocket clearances is not always practical in bearing design. Past experiences indicate that high ball speed variation-induced ball/ball pocket loads (as aggravated by uneven transfer from certain self-lubricating composite retainers) required not reducing but increasing the ball/pocket clearances. Shouldn't the future, upgraded versions of the solid-lubricated DREB concentrate on more complex retainer geometries, including segmented or biased retainer configurations and incorporating composite wear rate, transfer film formation tendencies and film traction into a holistic retainer design?
3. Could the initial position of each bearing component be determined more accurately with solid-lubricated bearings than with the liquid-lubricated counterparts? Or is this a moot point, when dynamic programs are run long enough to predict the equilibrium behavior of a spinning bearing?

It is my sincere hope that additional work, as exemplified by Dr. Gupta's efforts will continue in the future and will see the light of publication. Soon we will have sufficient solid-lubricated rolling element bearing test data to compare theory with practice. A more thorough critique of papers like the present one must await those test results and the simultaneous upgrading of the models themselves.

AUTHOR'S CLOSURE

The author is pleased to see that the predicted stability of the cage agrees well with the simplified stability criterion, due to Mr. Kannel, in Eq. [A1]. However, two points must be remembered. First, Eq. [A1], even with the correct inputs, is no more than a speculation of cage stability and a refinement of the stability criterion will require further studies to investigate the interrelationships between the materials and the kinematic factors influencing the overall cage motion. Second, the paper does not imply that the bearing behavior is insensitive to the ball/race traction. In fact, the ball/race traction-slip relation will be a key element in determining the optimum ball/cage and cage/race clearance for acceptable performance of the bearing under prescribed operating conditions.

As pointed out by Mr. Gardos, it is true that there will be a certain threshold slope of the traction curve which will define the range over which a solid-lubricated bearing can be designed. Also, over a certain range of slopes, the bearing behavior may very well be fairly insensitive to the actual magnitude of the traction-slip slope and it may only be necessary that the slope be positive. This is discussed in more depth in another paper (*C1*). The treatment of complex cage geometries, including the unbalanced, biased, and segmented cage designs, is a very simple task in the modular computer codes such as DREB. However, in the absence of adequate experimental validation even for the simplest design, the introduction of further sophistications in DREB may not be justifiable. The author is currently instrumenting the fundamental cage motions and comparing them with DREB predictions. Following such a validation, DREB can be further enhanced to consider much more complicated bearing designs and operating environment. With regard to the selection of initial conditions, it should be understood that by definition of a stable integrating algorithm, the final steady-state solution will be insensitive to the perturbations in the initial conditions. However, a careful selection of the initial conditions may greatly reduce the computational effort required to obtain steady state solutions. Finally, in response to Mr. Gardos' comments, it must be clarified that DREB does not "iterate" a large number of nonlinear algebraic equations, as done in equilibrium solutions; instead, it solves a large number of ordinary differential equations using an explicit integrating algorithm. Thus, the required computer time is related to the number of function evaluations or calculation of derivatives over the time domain of integration. Practical limitations of computer time are really dependent on the computer systems used. For example, some of the high-speed machines hardly demonstrate any computer time problems. With some of the slower machines, however, computer time can be a real problem. Subsequent to the work reported in this paper, the author has investigated certain constraints which can greatly reduce the computational effort and provide reasonable performance simulations over a large number of shaft revolutions (*C2*). It is expected that such advancements, along with the rapidly advancing computer technology, will, indeed, prove that any computer time related problems are only temporary, as discussed by Mr. Gardos.

REFERENCES

(*C1*) Gupta, P. K., "Simulation of Torque in High-Speed Solid Lubricated Gyro Ball Bearings," presented at 1981 Applied Modelling and Simulation Conference, September 7–11, 1981, Lyon, France.

(*C2*) Gupta, P. K., "Interactive Graphic Simulation of Rolling Element Bearings, Phase I: Low Frequency Phenomena and RAPIDREB Development," Air Force Report AFWAL-TR-4148, Contract No. F33615-80-C-5152, September 1981.